***NATURE OF A SYSTEM OF LINEAR EQUATION***

***INTRODUCTON***

* **In Engineering Mathematics, solution of simultaneous equations is very important. In this chapter we shall study the system of linear equations with emphasis on their solution by means of determinants.**
* **Definition of Matrix:- A matrix is a rectangular array of numbers arranged in rows and columns.If m-n number are arranged in a rectangular array of m rows and n columns, it is called a matrix of order m by n(written as m x n) or a matrix of dimension m x n or a matrix of type m x . A matrix of order m x n is usually written as follows :**

The number etc. Constituting a matrix are called the elements or entries of the matrix

**Various Type of Matri**x:-

1. **Square Matrix:-** A matrix in which the number of rows is equal to the number of columns, called square matix e.g.
2. **Diagonal Matrix:-** A square matrix is called diagonal matrix, if all its non-diagonal elements are zero e.g.
3. **Unitor Identity Matrix:-** A square matrix is called a unit matrix if all jthe diagonal elements are unity and non-diagonal elements are zero e.g.

1. **Null Matrix or Zero Matrix:-** Any matrix, in which all the elements are zeros, is called a zero or Null matrix e.g.

1. **Symmetric Matrix:-** A square matrix will be called symmectric, if for all values of I and j, i.e. aij=aji A`= A e.g
2. **Skew Symmetric Matrix:-** A square matrix is called Skew-symmectric Matrix, if (1) aij=-aji for all values of I and j or A`=-A

(2) All diagonal elements are zero e.g.

1. **Transpose of Matrix:-** If in a given matrix A, We interchange the rows and the Columns, the new matrix obtained is called the transpose of the matrix A and is denoted by A` or AT  e.g.

A= , A`=

1. **Echelon Matrix:-** A matrix ‘A’ is said to be Echelon matrix if all zero rows of ‘A’ follow all non-zero rows of ‘A’ and the number of zero Preceding the first non zero element of a row increasing as we pass from row to row .

, ,

1. **Coefficient Matrix:-** A system with m number of linear equation and n number of unknown can be written as

a11x1+a12x2+………+a1nxn = b1

a21x1+a22x2+………+a2nxn = b2

**.**

**.**

**.**

Am1x1+am2x2+………+amnxn = bm

Where x1 , x2,……, xn are the unknown and number a11 , a12,…….amn are the coefficients of the system. The coefficient matrix is mxn order matrix with elements.

1. **Augmented Matrix:-** In linear algebra, an augmented matrix is a matrix which obtained by appending the columns of two given matrices, usually for the purpose of the performing same elementary row operations on each of the given matrices . Suppose A be an Augmented matrix then this is denoted by

Given the matrices A & B, where

, B=

The augmented matrix (A/B) is written as

(A/B) =

This type of matrix is very useful when solving the system of linear equations.

1. **Rank of Matrix:-** ‘A’ be a matrix, a natural number r is said to be rank of ‘A’ denoted by R(A) if
2. There is at least one rth order non-singular square submatrix of ‘A’
3. Every square sub matrix of ‘A’ of order greater than r is singular

Example:- Find the rank of the given matrix

A

R2-3R1

R3-2R1

R4-R1

A   
 This is the required matrix of ‘A’ and R(A)=3

R3-6/7R2

R4-3/7R2

**System of an Equations:-** A linear equation in a unknowns x1,x2,….....xn is an equation of from a1x1+a2x2+………+anxn=b ………(1)

In the above equation, if b=0 then it is called a homogeneous linear equation.In contrast, (I) is called a non homogeneous linear equation.

Consider a set of m non-homogeneous linear equations in n unknowns:

A11x1+a12x2+……….+a1nxn=b1

A21x1+a22x2+……….+a2nxn=b2

… …………… …………….. ……………

… …………… …………….. ……………

Am1x1+am2x2+……….+amnxn=bm

If at least one set of values of x1,x2,……….xn can be found satisfying all the equations, then the equations are said to be consistent. If no such set be possible, then the equations are said to be inconsistent

In the former case, the values of the unknowns for which all the equations are satisfied are said to constitute the solutions of the equation.

System of Equations

Inconsistent

[R(A)R()]

Consistent

[R(A)=R()]

Infinite

[R(A)=R() no of Unknown]

Unique

[R(A)=R()]= no of Unknown

**Example:-** Determine the Condition under which the equation.

x + y + z = 1

x + 2y – z = b

5x + 7y + az = b2

At which i) Unique , ii) Infinite , iii) no solution

A =

R2-R1

R3-5R1

Augmented matrix ( =

R3-2R2

**Case-1:-**

if a-1 = 0 , R(A) = 2

& b2-2b-3 = 0 R() = 2

Hence the system is Consistent type and infinite Solution

**Case-2:-**

If a-1 = 0 , R(A) = 2

& b2-2b-3 0 , R() = 3

Hence the system is Inconsistent type and there is no solution

**Case-3:-**

If a-1 0 then R(A) = 3

& b2-2b-3 0 then R() = 3

Hence the system is consistent type and there is unique solution

Example:- For what value of A the following equation is given bellow

x + y + z = 1

2x + y + 4z = k

4x + y + 10z = k2

At which i) Unique , ii) Infinite , iii) no solution

A =

R3-2R2

R2-2R1

R3-4R1

Augmented matrix ( =

Where R(A) = 2

**Case : 1**

If k2-3k+2 0 then R() = 3 , R(A) = 2

R(A) R() , so the system is inconsistend type and there is no solution.

**Case : 2**

If k2-k+2 = 0

Then R() = 2 , R(A) = 2

R(A) = R() , Hence the system is consistent type and there is Infinite solution.